An adaptive finite element spectral wave model

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Interest in wave prediction grew during the Second World War, driven by the need to predict the random sea state for landing operations. The work that followed led to the introduction of the wave spectrum and soon after spectral wave modelling emerged. Unlike phase-resolving models which explicitly reconstruct the sea surface elevation in space and time, spectral (or phase-averaged) models calculate the energy density spectrum from which most of the important statistical parameters can be derived. This is significantly less computational expensive, making this framework ideal for large scale applications such as forecasting, climate studies and resource assessments, with an impact on a wide range of sectors, such as coastal engineering, offshore engineering, marine renewable energy etc.

The governing equation, called the energy balance equation, is a 5-dimensional system dependent on geographical space (x, y), spectral space (directions, frequencies) and time. Even though this framework has made global wave predictions possible, the multidimensional nature sets strict limits on the accuracy of the solutions, especially on large scale simulations.

For example, a known problem associated with this restriction is the 'Garden Sprinkler Effect' (GSE) which refers to the spurious separation of a propagating wave field into the discretised angular and frequency bins.

This work focuses on the development of an adaptive finite element spectral wave model, aiming to increase the range of possible resolutions in spectral wave modelling, while keeping the computational costs low.

Owing to the hyperbolic nature of the system a continuous finite element expansion in geographical space is problematic.

To achieve a stable and computationally efficient discretisation a sub-grid scale finite element formulation is thus developed, which combines the benefits of continuous and discontinuous finite elements, i.e. low computational cost, and accuracy and stability respectively. For the angular discretisation, Haar wavelets are implemented. These are piecewise constant hierarchical basis function with compact support, making the application of anisotropic adaptive resolution in the angular phase-space possible.

The code is verified against standard test cases such as current and depth induced shoaling and refraction.

A new framework for applying angular adaptivity is developed and various convergence plots are presented to show the benefits of adaptivity both in space and angle.

Finally a GSE test case is presented to show how angular adaptivity can be used to resolve it without prohibitively increasing the computational costs.