# Alternative Transport Scheme for Mass Conservation in ELCIRC Model

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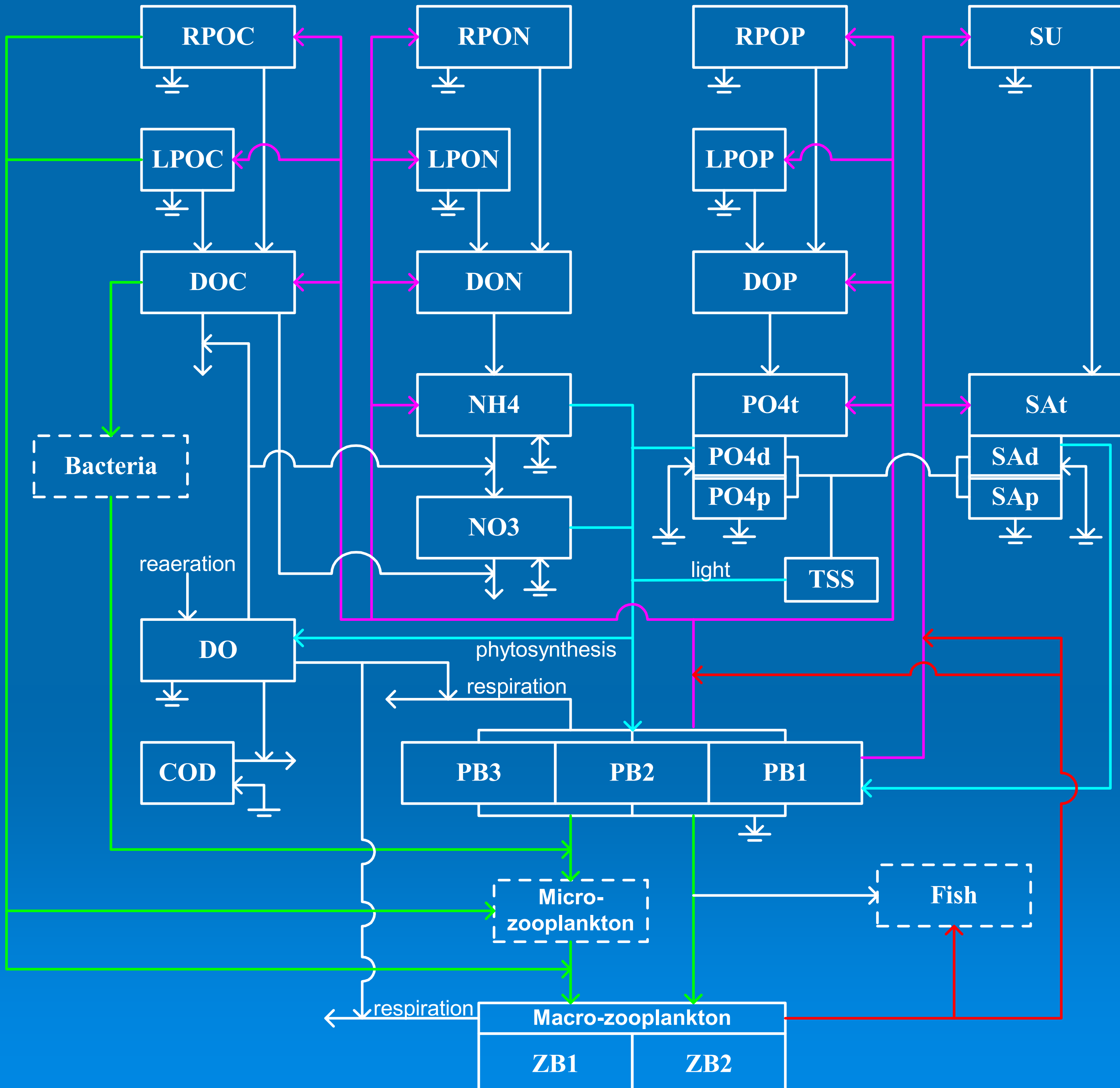
# Danshuei River estuary


According to Wang et al. (2004)

- Short residence time (≈1~3 days)
- Significant fraction of anthropogenic pollutants may exert their effects in the coastal waters outside the estuary

# Objective

- To evaluate the impacts of wastewater discharges on the aquatic environments in the coastal waters as well as in the estuary
- A water quality model is being developed utilizing the hydrodynamic information computed by the ELCIRC 3-D hydrodynamic model
  - The water quality model is internally coupled with the ELCIRC, using the same grid for spatial resolution
  - 22 state variables are considered in the water quality model



RPOC
RPON
RPOP
SU
LPOC
LPON
LPOP
DOC
DON
DOP
NH4
PO4t
SAt
Bacteria
PO4d
PO4p
SAd
SAp
NO3
reaeration
light
TSS
DO
phytosynthesis
respiration
COD
PB3
PB2
PB1
Micro-
zooplankton
Fish
respiration
Macro-zooplankton
ZB1
ZB2

# Mass-balance eqn.

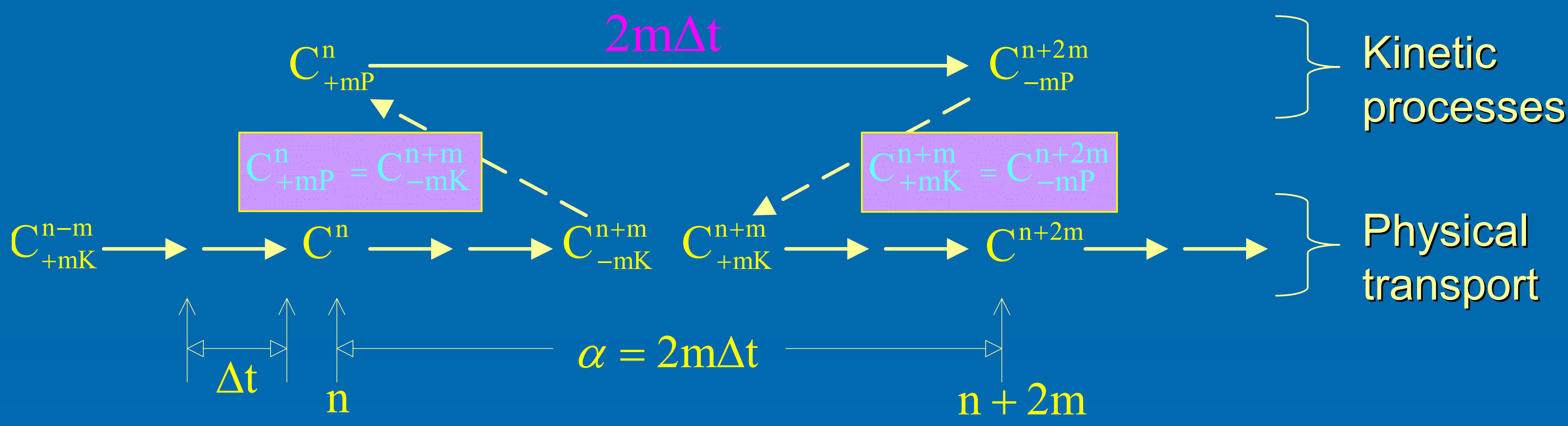
- General form

$$\frac{\partial (VC)}{\partial t} = \begin{bmatrix} physical\ \ transport \\ processes \end{bmatrix} + \begin{bmatrix} biogeochemical \\ processes \end{bmatrix}$$

- Decoupled
  - Physical transport eqn. $\frac{\partial (VC)}{\partial t} = \begin{bmatrix} physical\ \ transport \\ processes \end{bmatrix}$
  - Kinetic eqn. $\frac{\partial (VC)}{\partial t} = \begin{bmatrix} biogeochemical \\ processes \end{bmatrix}$

# Decoupled mass-balance eqn.

- Multi-step computation scheme for intra-tidal models (Park et al., 1998)




$$C^{n+m} = \frac{C^{n+m}_{-mK} + C^{n+m}_{+mK}}{2} \quad or \quad C^{n+m} = \frac{C^{n}_{+mP} + C^{n+2m}_{-mP}}{2}$$

# Physical Transport

- Finite volume/finite difference scheme is used, similar to CE-QUAL-ICM Model
- Transport fluxes in physical transport eqn. are formulated to be consistent with the continuity eqn. in ELCIRC

# The vertical-integrated continuity eqn.

$$P_i\left(\eta_i^{n+1}-\eta_i^n\right)+\theta\Delta t\left[\sum_{l=1}^{i34(i)} s_{i,l}\ell_{jsj}\left(\sum_{k=m_{jsj}}^{M_{jsj}}\Delta z_{jsj,k}^{n}u_{jsj,k}^{n+1}\right)\right]+(1-\theta)\Delta t\left[\sum_{l=1}^{i34(i)} s_{i,l}\ell_{jsj}\left(\sum_{k=m_{jsj}}^{M_{jsj}}\Delta z_{jsj,k}^{n}u_{jsj,k}^{n}\right)\right]=0$$

Computation of physical transport for cell $k$ is completed in 2 steps

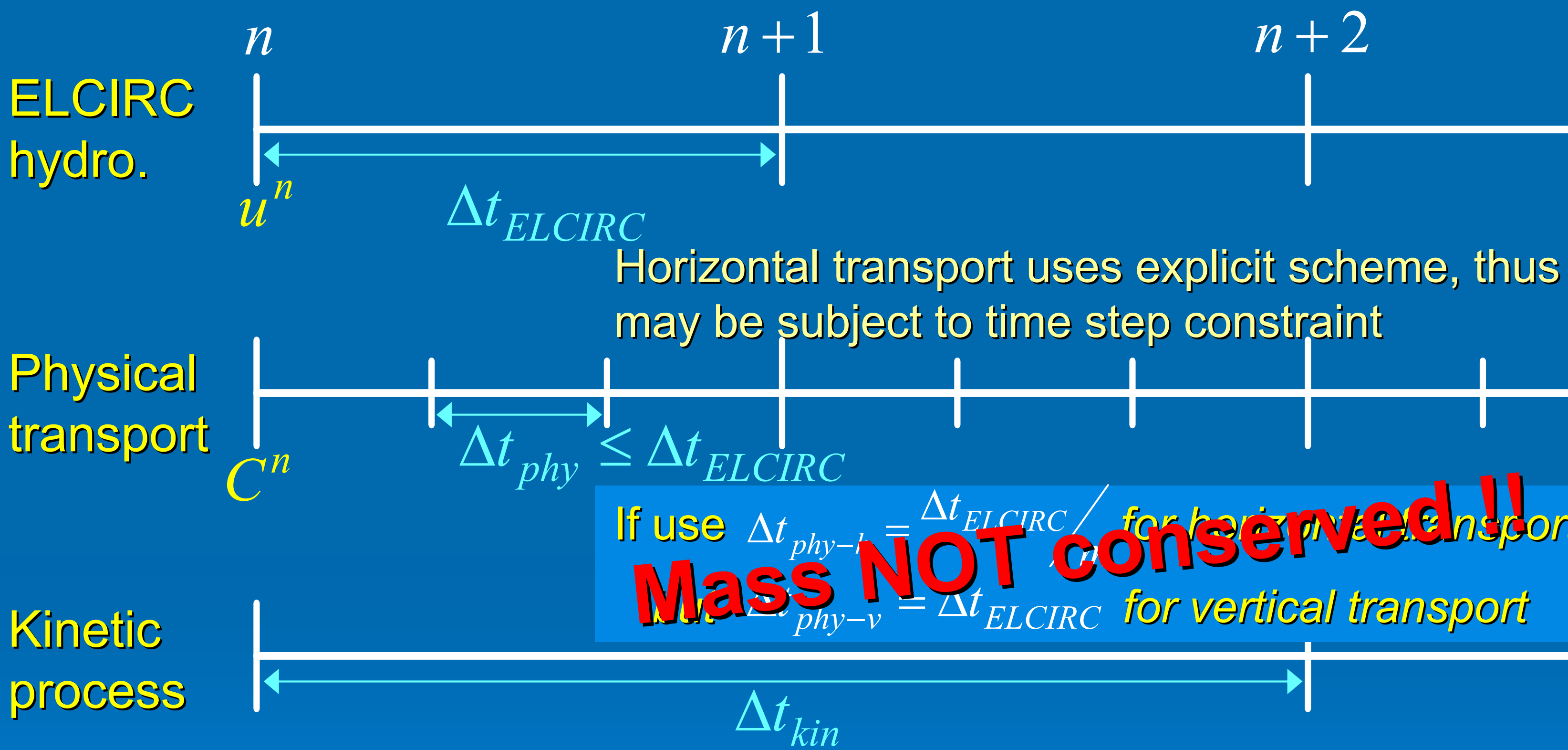
1. Explicit (horizontal fluxes and explicit part of vertical fluxes, use backward difference for horizontal fluxes)

$$C_k^{*}V_k^{n+1}=C_k^nV_k^n-\theta\Delta t\left[\sum_{l=1}^{i34(i)} s_{i,l}\ell_{jsj}\left(\Delta z_{jsj,k}^{n}u_{jsj,k}^{n+1}C_{u_1}^n\right)\right]-(1-\theta)\Delta t\left[\sum_{l=1}^{i34(i)} s_{i,l}\ell_{jsj}\left(\Delta z_{jsj,k}^{n}u_{jsj,k}^{n}C_{u_2}^n\right)\right]$$
$$-(1-\theta)\Delta t\left[A_i\left(w_{i,k}^nC_{k,k+1}^n-w_{i,k-1}^nC_{k-1,k}^n\right)\right]$$

2. Implicit (implicit part of vertical fluxes and vertical diffusive fluxes)

$$C_k^{n+1}V_k^{n+1}=C_k^{*}V_k^{n+1}-\theta\Delta t\left[A_i\left(w_{i,k}^{n+1}C_{k,k+1}^{n+1}-w_{i,k-1}^{n+1}C_{k-1,k}^{n+1}\right)\right]-\Delta t\left[A_i\left(\sum K_{cv}\frac{\partial C^{n+1}}{\partial z}\right)\right]$$

# Computation sequence




Horizontal transport uses explicit scheme, thus may be subject to time step constraint

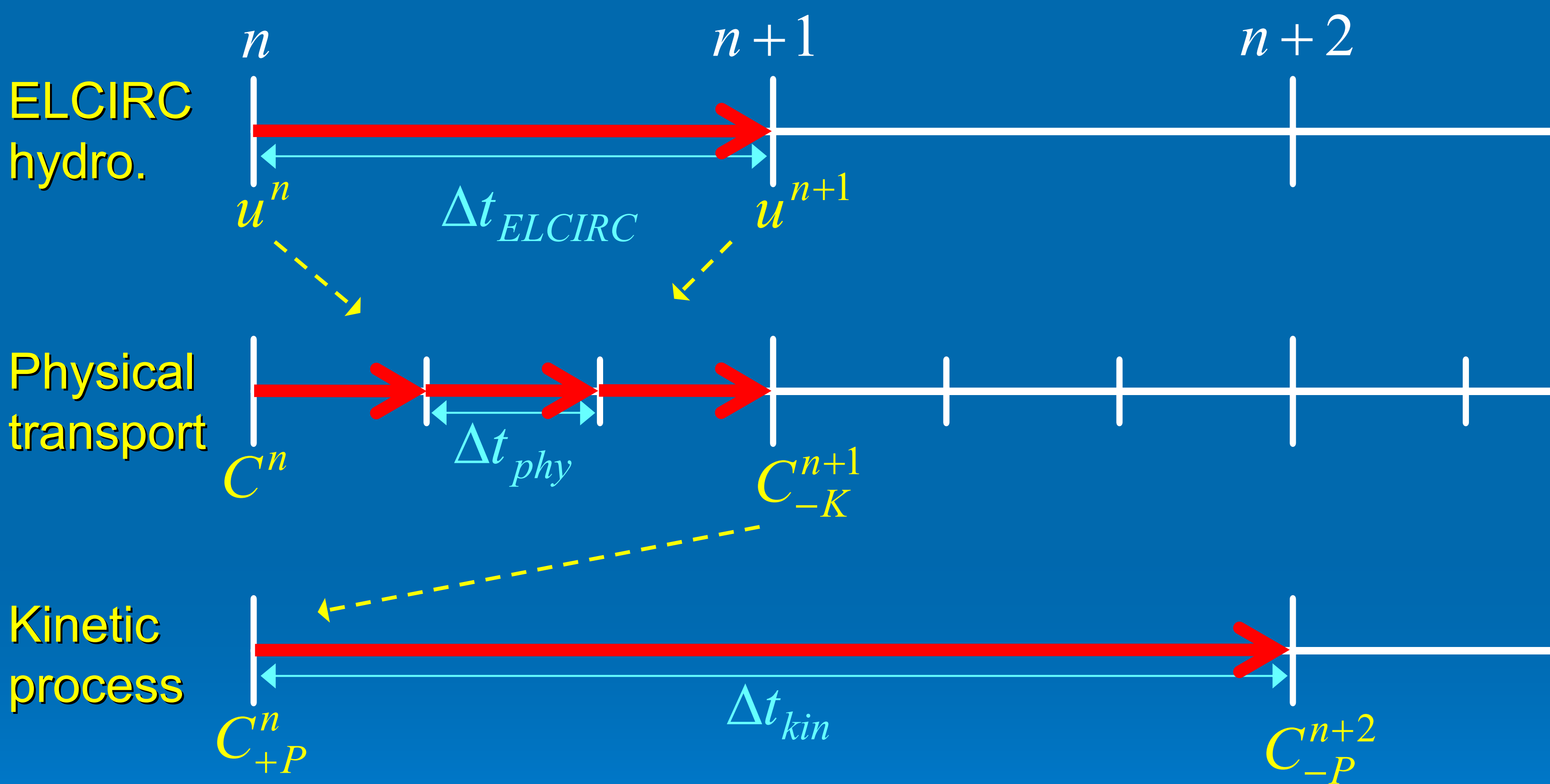
If use $\Delta t_{phy-h} = \Delta t_{ELCIRC} / m$ for horizontal transport

but $\Delta t_{phy-v} = \Delta t_{ELCIRC}$ for vertical transport

**Mass NOT conserved !!**

Time step of kinetic processes is user-specified, usually multiple of $\Delta t_{ELCIRC}$

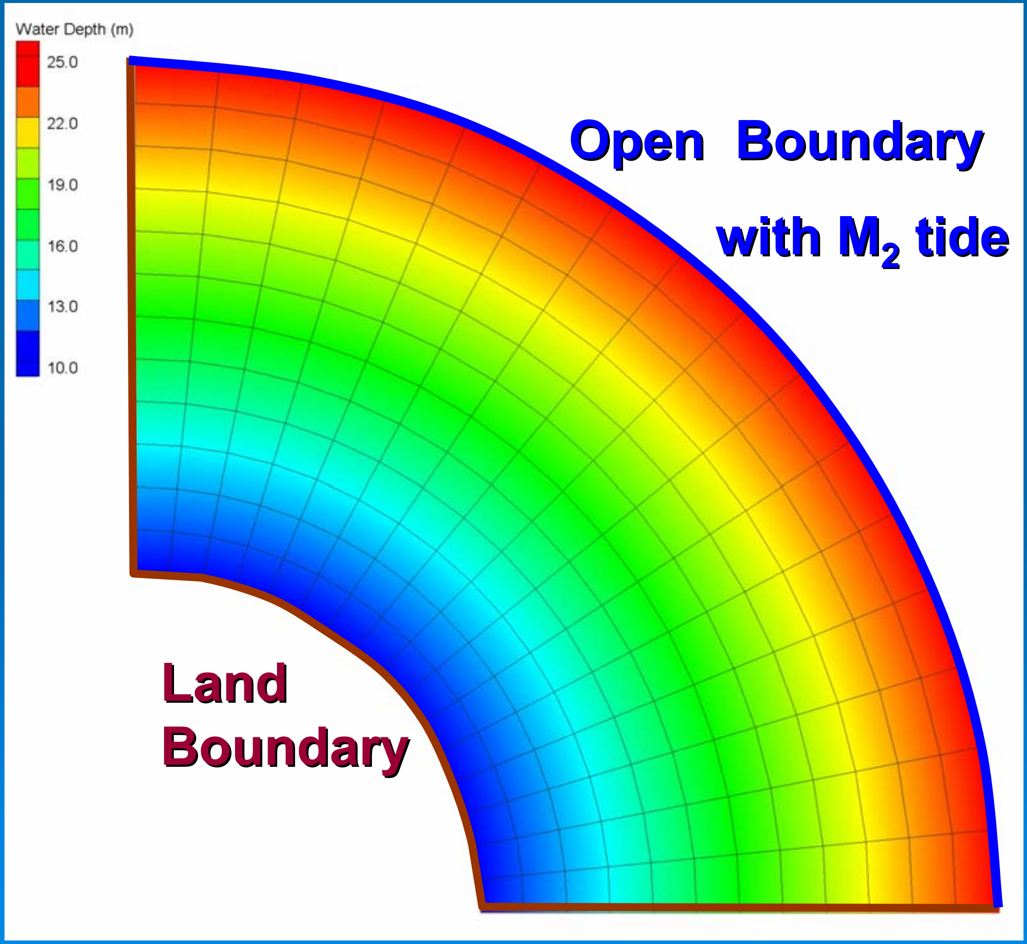
# Computation sequence




$$\Rightarrow \quad C^{n+1} = \frac{C^{n}_{+P} + C^{n+2}_{-P}}{2}$$

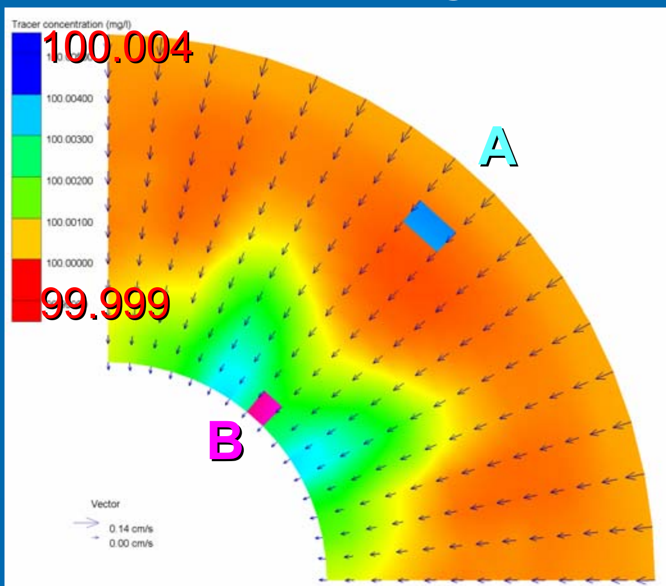
# Test case #1 — uniform conc.

- Quarter annulus
- Has been compared with analytical solutions for tidal propagation
- Initial *conservative tracer* C=100 mg/l everywhere
- At open boundary C=100 mg/l at all time

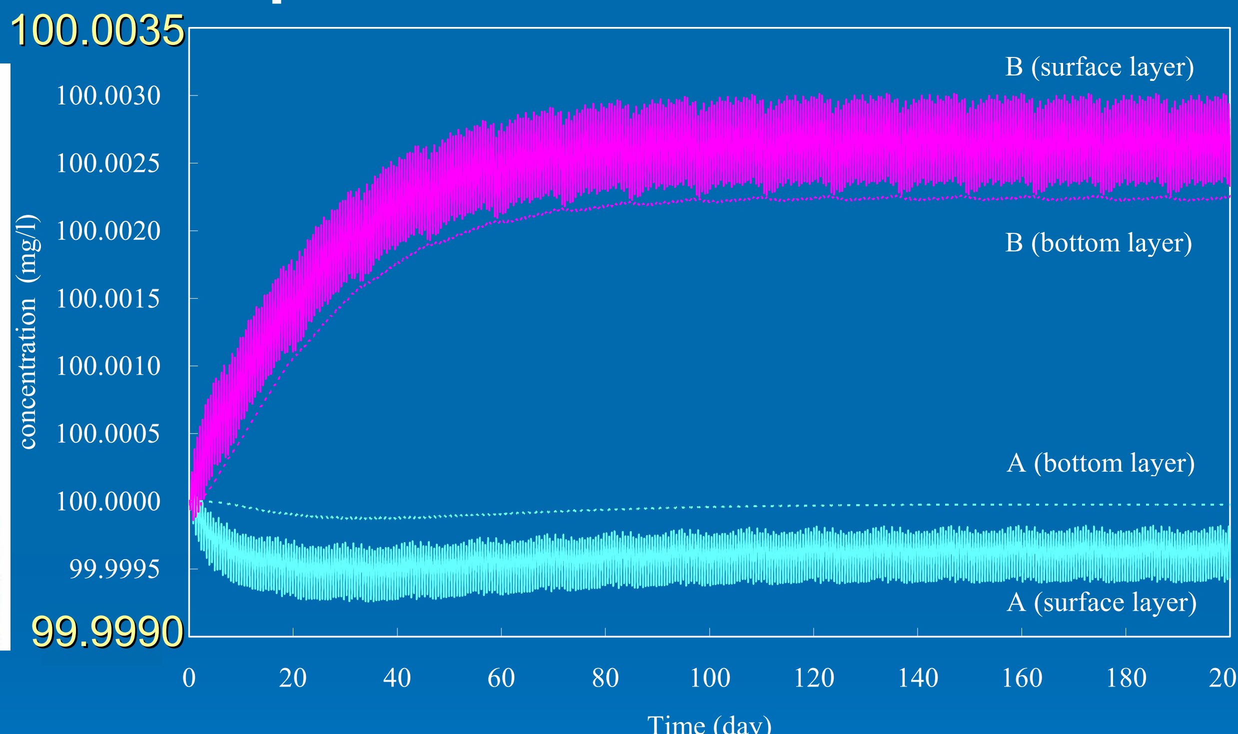


# Tracer concentration

**Spatial dist. at t= 60 day**




**Temporal vari. at elements A and B**



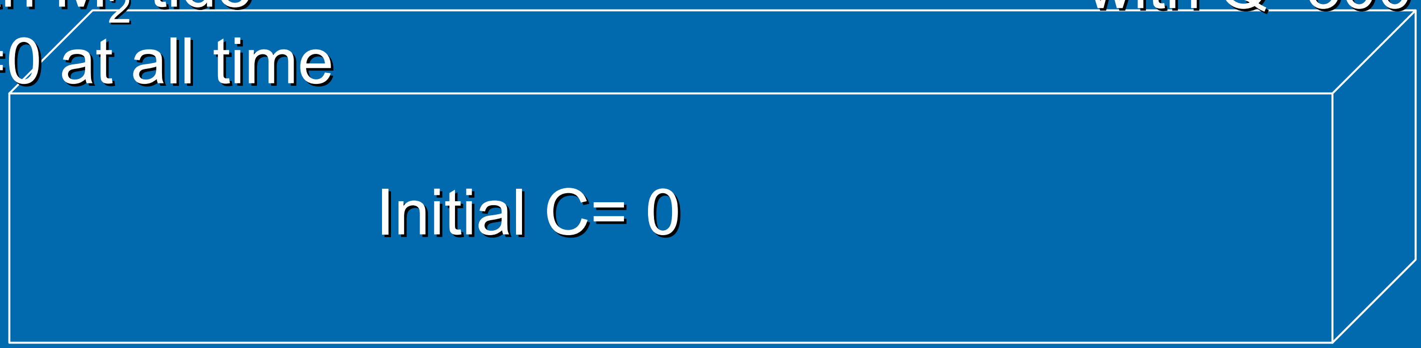

- Overall concentrations range from 99.99881 to 100.00508 mg/l

(i.e., -0.00119 ~ +0.00508 % error)

# Test case #2 — tracer release

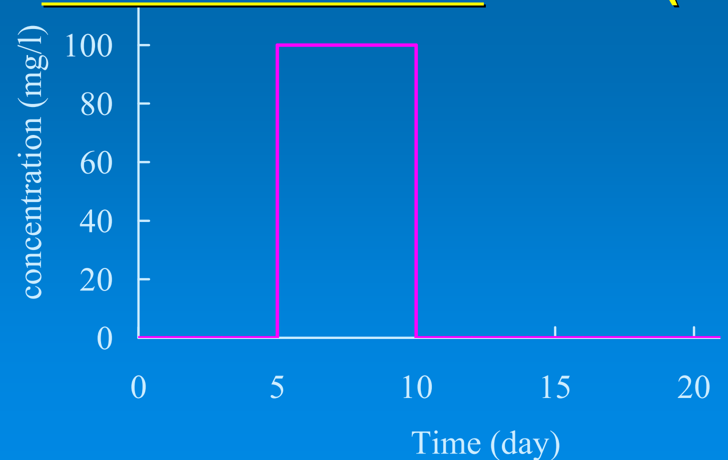
Open boundary
with $M_2$ tide
C=0 at all time

River boundary
with Q=300 $m^3/s$




Rectangular channel
L= 170 km
B= 5 km
D= 10 m

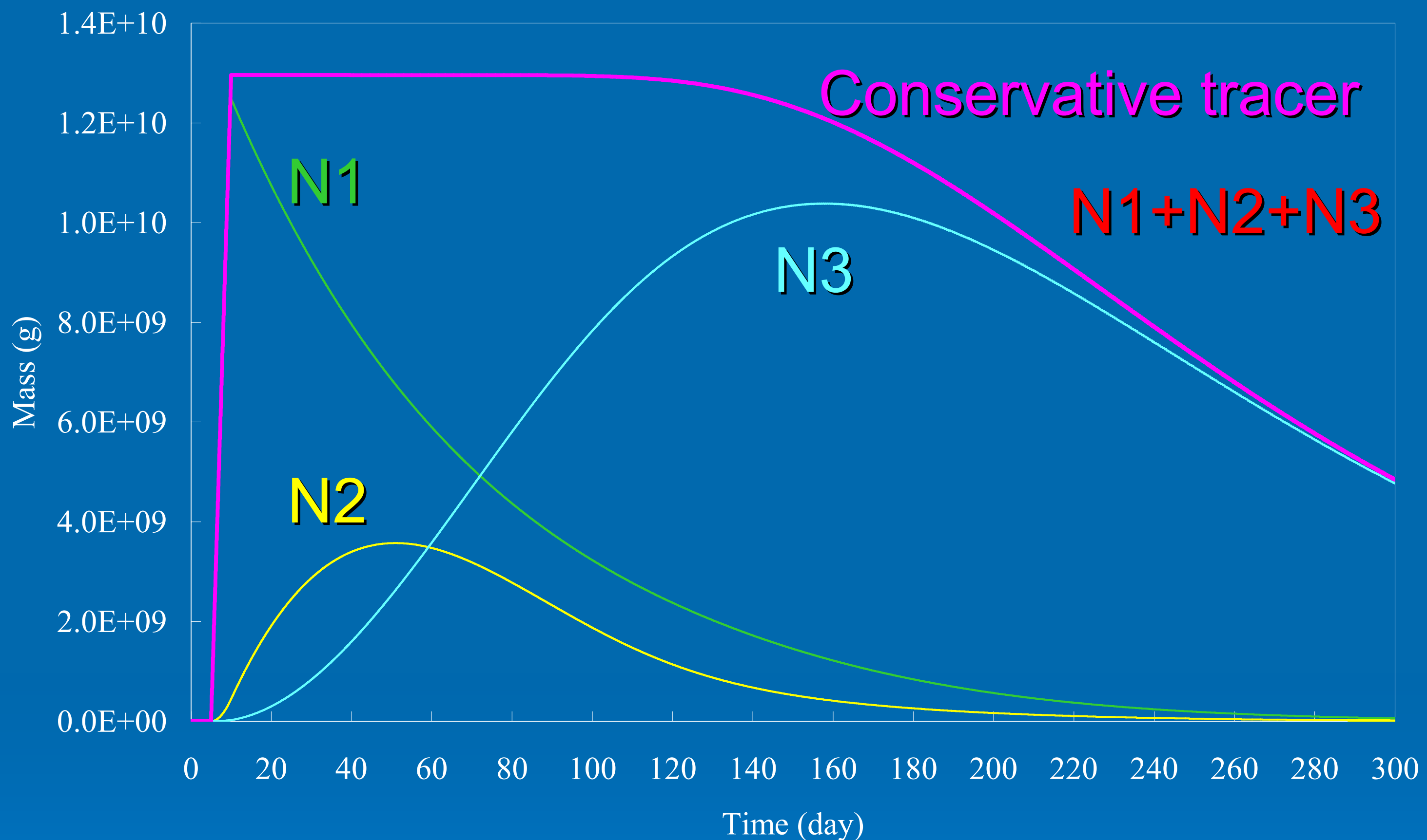
**From *t= 5~10 day*, river discharge carries one *conservative* tracer and one *non-conservative* tracer (N1)**




**Note: Non-conservative tracer**

**N1 → N2 → N3**

# Total mass of tracer within model domain

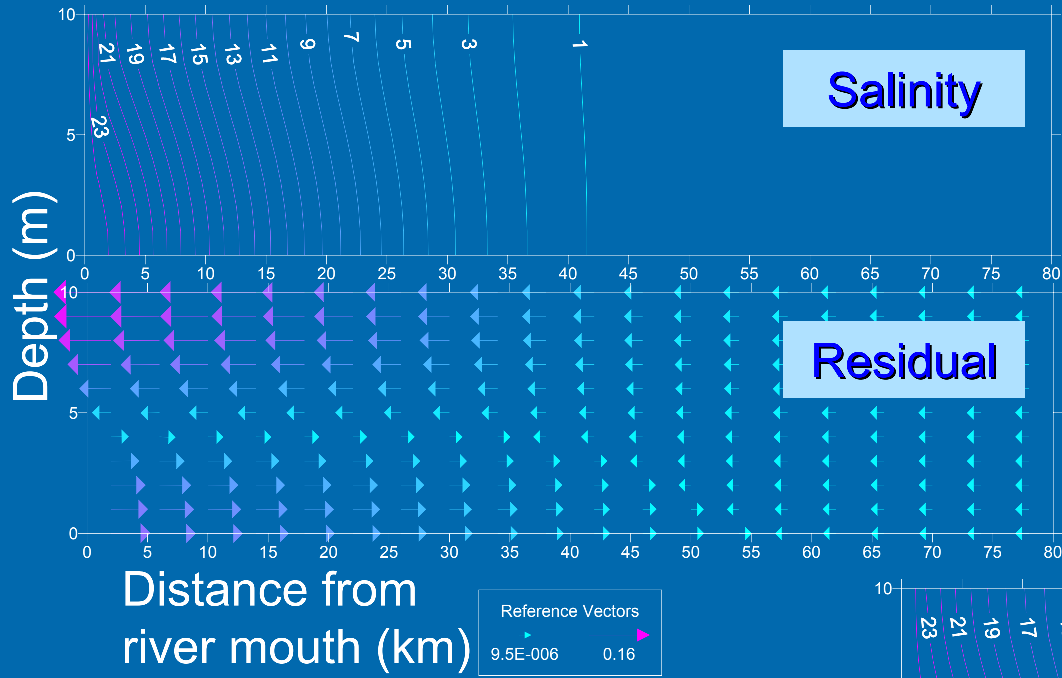



- Error between *input* total mass and the *actual* total mass of *conservative tracer* at t=10 day is 0.000375 %
- Total mass of N1+N2+N3 is the same as that of *conservative tracer*

# Conclusions

- The finite-volume/finite-difference form of physical transport eqn. in water quality model must be carefully formulated to be consistent with the continuity eqn. in hydrodynamic model, otherwise, mass will not conserve.
- Results of the test cases are reasonable and mass-conservative, which assures us to continue developing the water quality model.

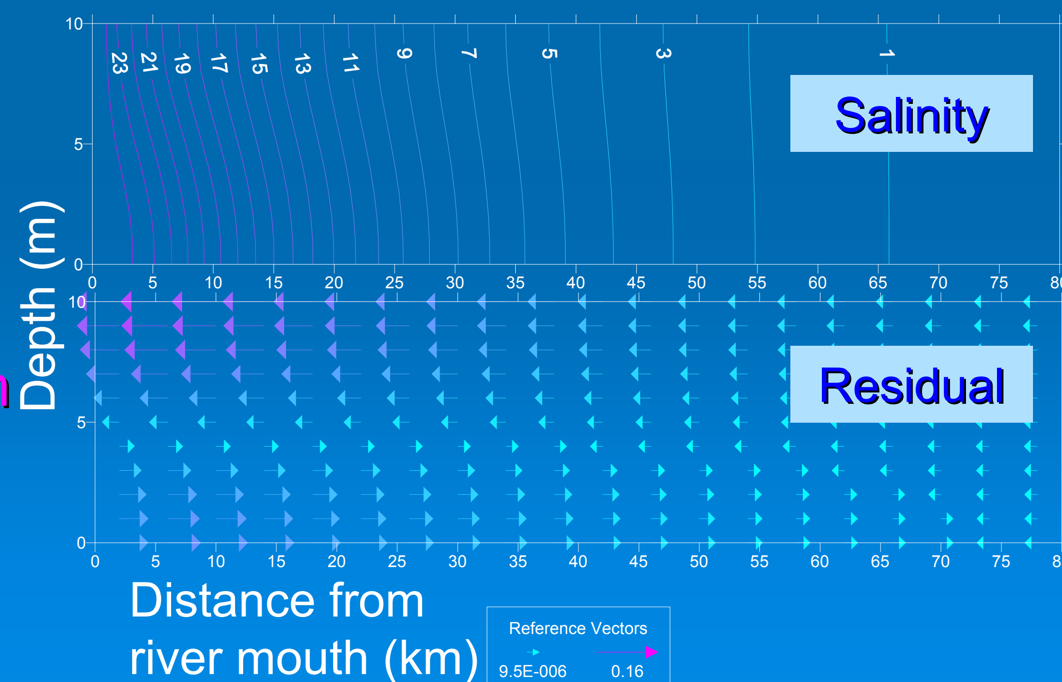
# ELCIRC transport vs. Upwind transport




## ELCIRC transport

Salinity intrusion ≈ 42 km

Max. upstream residual ≈ 0.106 m/s




## Upwind transport

Salinity intrusion ≈ 66 km

Max. upstream residual ≈ 0.066 m/s

## Could OHSU modify the continuity eqn. in ELCIRC to simulate the volume discharge from ocean outfall ?